Radius of curvature limitation and accuracy of a surface profile measurement

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31 July, 2001

Abstract

This document describes the ability of various types of surface topography measuring instruments to determine the radius of curvature if the surface is expected to have a spherical figure. These abilities are classified as limitations of radius measurement ("vertical" or dynamic range) and accuracy of radius measurement. The discussion is limited to a two-dimensional representation of the surface, also known as a surface profile.

Keywords

radius of curvature accuracy, surface profile measurement

Introduction

This topic applies to any kind of surface profiler, but especially to scanning profilers such as the long trace profiler (LTP). Methods for measuring the RoC of a surface are discussed in Malacara[1]. Deducing the radius of curvature from the surface topography acquired from phase shift profilers has been addressed[1,2], and nulling methods exist for getting an accurate number for the radius of curvature[3,4]. A scanning profile is here meant as a profiler with a constant velocity scanning carriage or probe motion, rather than a series of step-and-repeat stationary measurements. The LTP is useful for measuring optical surfaces whose figure ($L > 10$ mm) is in a spatial domain difficult for other instruments to accurately measure. One attribute that is often used as a single-number description of the surface is its radius of curvature (RoC). The RoC has a value R which is the distance from a purely spherical surface to the center of the sphere. In the case of a single line profile measurement, R is the radius of a circle.

Instruments have been made exclusively for accurately measuring relatively short RoC[1,4]. Perhaps the simplest of these uses a spherometer, which interprets the RoC as a sag of the surface along a chord of length L on the spherical surface. For the case of R >> L >> sag,

$$R = \frac{L^2}{8\ \text{sag}} \quad . \tag{1}$$

This is a useful way to check for first-order errors in the calculation of R when a surface height profile is given over some surface length L.

Short RoC limitation and accuracy

For a spherical mirror with radius of curvature R and a slope-measuring instrument with a maximum slope measurement range of ϕ_{max} , the longest profile that can be measured is

$$L_{max} = R \cdot \phi_{max} \quad . \tag{2}$$

If the spatial sampling interval is dx, then the maximum number of data points that can be taken for a measurement scan is

$$N_{max} = \frac{L_{max}}{dx} = \frac{R \cdot \phi_{max}}{dx} \quad . \tag{3}$$

For typical LTP parameters of ϕ_{max} = 0.01 and dx = 0.001m, Nmax = R[m] · 0.01 / (0.001m) = 10 R[m] /m. In other words, the LTP can measure at most 10 data points on a mirror with R = 1m.

Error from jitter in high-curvature surfaces

In general, accuracy in determining deviation from a short RoC best-fit circle is limited by lateral sampling interval uncertainty (jitter) as shown in Figure 2.1. For a sampling interval of dx with a position precision of Δx, the maximum peak-to-valley height variation Δh_{pv} will occur where the slope is greatest. If a perfectly spherical surface is approximated as a normalized parabola (vertex at the origin and no residual piston or tilt), then the ideal profile will be

$$h(x) = \frac{1}{2R} x^2 \quad , \tag{4}$$

and the maximum discrepancy will be at L/2 (the edge of the mirror) where the slope is greatest. The peak-to-valley variation there will be

$$\Delta h_{pv} = \frac{1}{2R}\left(2x\big|_{L/2}\ \Delta x\right) = \frac{L}{2R}\ \Delta x \quad . \tag{5}$$

For height-measuring profile scanning instruments which scan the surface with speed v, total jitter is the sum of spatial uncertainty (where the probe is on the surface) and temporal uncertainty (when the data is taken from the probe). Thus the short RoC accuracy is limited by the error

$$\Delta h_{pv} = \frac{L}{2R}\left(\Delta x + v \cdot \Delta t\right) \quad , \tag{6}$$

where Δt is the precision of knowing exactly when the data was acquired and can have a value between the data acquisition's sampling clock interval and dx / v. In this case Δh_{pv} is the peak-to-valley height variation after the best-fit sphere would be removed from any perfectly spherical surface measurement.

For slope-measuring instruments the ideal profile is the x derivative of the parabolic height profile in Equation (3):

$$s(x) = \frac{x}{R} , \tag{7}$$

and the slope change magnitude Δs_{pv} between adjacent samples is

$$\Delta s_{pv} = \frac{1}{R} \Delta x . \tag{8}$$

This error will vary only with position uncertainly Δx. For scanning instruments

$$\Delta s_{pv} = \frac{1}{R} (\Delta x + v \cdot \Delta t) . \tag{9}$$

For a sporadic distribution of jitter (every 1 out of N samples) on an otherwise perfect surface, the overall root-mean-square (rms) slope variation is

$$\Delta s_{rms}{}^2 = \sum_{i=1}^{N} s_i{}^2 \ / \ N = \left[\frac{1}{R} (\Delta x + v \cdot \Delta t) \right]^2 \ / \ N . \tag{10}$$

Therefore,

$$\Delta s_{rms} = \frac{1}{R \sqrt{N}} (\Delta x + v \cdot \Delta t) . \tag{11}$$

<u>An example for a scanning profiler</u>

As a case in point, measurements of a spherical mirror (R = 11m) were made with an LTP that was using a line-scan camera not suited for precision time-domain image capturing. The measurements were out-of-specification and contained sparse impulses (about 1 in 25 data points). The observed impulses were not repeatable and in slope instead of height, indicating that these impulses could be from variations in the camera's image capture timing. Although Δx was 0.0005 mm, $v \cdot \Delta t$ was about 0.2 mm, as seen from intensity patterns during the conversion to slope. If the temporal uncertainty were insignificant, then we would expect errors from spatial uncertainty in the order of

$$\Delta s_{pv} = \Delta s_{rms} = \frac{1}{R} \Delta x = 0.045 \, \mu\text{rad} . \tag{12}$$

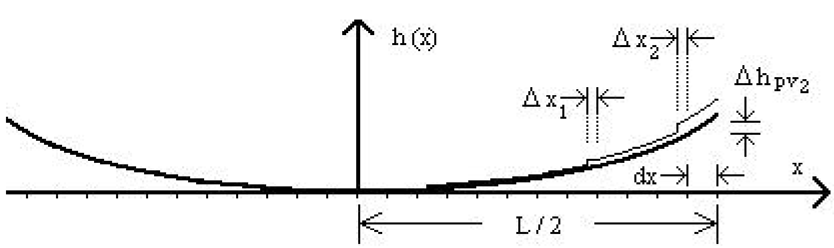

Figure 1. True parabolic surface (bold curve) and surface profile with two instances of jitter (light segments).

If the temporal jitter component were taken into account, however, the expected error becomes

$$\Delta s_{pv} = \frac{1}{R} (\Delta x + v \cdot \Delta t) = 18.1\ \mu\text{rad.} \tag{13}$$

and

$$\Delta s_{rms} = \frac{1}{R\sqrt{N}} (\Delta x + v \cdot \Delta t) = 3.6\ \mu\text{rad.} \tag{14}$$

Actual LTP measurements using this camera resulted in slope errors of between 2.5 and 4.5 µrad rms, very close to the expected error. After measuring the same mirror with a more trustworthy camera, resulting slope errors were between 0.4 and 0.7 µrad; these variations are mostly from limitations of the metrology environment.

Long RoC limitation and accuracy

There is no inherent limitation on largest RoC measurements, but the largest practical RoC should be determined for any particular instrument and its environment.

In any instrument, RoC accuracy reaches its limit when height measurement variations are about equal to the spherical sag. Figure 2 shows the variables from a surface measurement which introduce error into the calculation for R. We differentiate Equation (1) to get

$$\Delta R = \frac{L^2}{8} (sag)^{-2}\ \Delta sag \quad , \tag{15}$$

and from Figure 2 we see that $\Delta sag = 2\ \Delta h$. Using Equation (1) again,

$$\Delta R = \frac{L^2}{8} \frac{2\ \Delta h}{sag^2} = \frac{L^2}{4} \Delta h \frac{64\ R^2}{L^4} \quad , \tag{16}$$

or

$$\Delta R = \frac{16\ R^2}{L^2} \Delta h \quad . \tag{17}$$

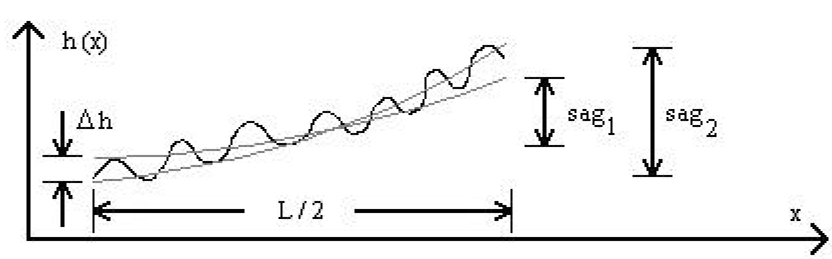

Figure 2. Radius calculation error from slope measurement error. Δsag = sag2 – sag1.

Examples for an interferometer

Consider a typical phase shift interferometry microscope (PSIM), for which $\Delta h = 1.0e\text{-}9$ m and $L = 1.0e\text{-}3$ m. The limiting accuracy for measuring a surface with $R = 1$ m will be $\Delta R = 16$ mm.

For a typical large-aperture phase shift interferometer, $\Delta h = 30e\text{-}9$ m and $L = 0.150$ m. The limiting accuracy for measuring a surface with $R = 100$ m will be $\Delta R = 0.2$ m; with $R = 1000$ m will be $\Delta R = 20$ m.

Example for an LTP

For a slope-measuring instrument the variation is expressed as slope: $\Delta h = dL\ \Delta s$, where dL is the spatial period with the greatest Δh. Then

$$\Delta R \quad = \quad \frac{16\ R^2}{L^2}\ dL\ \Delta s \quad . \tag{18}$$

As an example for the LTP, if the instrument has an rms slope accuracy of 0.354 µrad, then approximately $\Delta s = 1.0$ µrad pv. A typically worst case value for dL is about 156 mm. For a mirror with length $L = 0.5$ m,

$$\Delta R \quad = \quad \frac{16\ R^2}{0.25\ m^2}\ (0.156\ m)\ (1e\ \text{-}6) \quad = \quad 10e\ \text{-}\ 6\ R^2 \quad . \tag{19}$$

Then, for $R = 10$ m, $\Delta R = 0.001$ m ;
for $R = 100$ m, $\Delta R = 0.100$ m;
for $R = 1000$ m, $\Delta R = 10$ m.

Conclusion

Two limits for measuring radius of curvature of an optical surface on profiling instruments have been discussed, and calculation examples have been given in order to clarify the argument for the short RoC and long RoC limits.

It should be pointed out that instrumental resolution is not the limiting factor, nor does repeatability have anything to do with radius calculation of an acquired surface contour or profile. The accuracy in determining RoC is a function of both instrumental accuracy and the degree to which the surface is a perfect sphere within the field of view or instrumental aperture L. For scanning profilers, temporal jitter degrades the instrumental accuracy which reduces the accuracy of measuring the RoC.

Acknowledgments

We thank Peter Takacs for insights from a conversation about short RoC accuracy. We also thank Wayne McKinney for radius measurement discussions. The authors also thank Tom Tonnessen and Howard Padmore for their support of radius measurement investigation.

This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U. S. Department of Energy, Under contract No. DE-AC03-76SF00098.

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